

$$\textcircled{1} \quad \cot \left(\frac{B-C}{2} \right) = \frac{(b+c)}{(b-c)} \tan \frac{A}{2}$$

$$= \frac{(b+c)}{(b-c)} = \frac{\cot \left(\frac{B-C}{2} \right)}{\tan \frac{A}{2}}$$

$$= \frac{\frac{1}{k} \sin B + \frac{1}{k} \sin C}{\frac{1}{k} \sin B - \frac{1}{k} \sin C} = \frac{\cot \left(\frac{B-C}{2} \right)}{\tan \frac{A}{2}}$$

$$= \frac{\sin B + \sin C}{\sin B - \sin C}$$

$$= \frac{2 \sin \left(\frac{B+C}{2} \right) \cos \left(\frac{B-C}{2} \right)}{2 \cos \left(\frac{B+C}{2} \right) \sin \left(\frac{B-C}{2} \right)}$$

$$= \tan \left(\frac{B+C}{2} \right) \cot \left(\frac{B-C}{2} \right)$$

$$= \frac{\tan \left(\frac{B+C}{2} \right)}{\tan \left(\frac{B-C}{2} \right)} \cot \left(\frac{B-C}{2} \right)$$

$$= \frac{\cot \left(\frac{B-C}{2} \right)}{\tan \frac{A}{2}}$$

$$\therefore \frac{\cot \left(\frac{B-C}{2} \right)}{\tan \frac{A}{2}} = \frac{(b+c)}{(b-c)}$$

$$\cot \left(\frac{B-C}{2} \right) = \frac{(b+c)}{(b-c)} \tan \frac{A}{2}$$

$$⑧ \quad a \cos \left(\frac{B-C}{2} \right) = (b+c) \frac{\sin A}{2}$$

$$= \frac{a}{(b+c)} = \frac{\sin A}{\cos \left(\frac{B-C}{2} \right)}$$

LHS

$$= \frac{1/k \sin A}{1/k \sin B + 1/k \sin C}$$

$$= \frac{\sin A}{\sin B + \sin C}$$

$$= \frac{\sin A}{2 \sin \left(\frac{B+C}{2} \right) \cos \left(\frac{B-C}{2} \right)}$$

$$= \frac{2 \sin A/2 \cos A/2}{2 \cos A/2 \cos \left(\frac{B-C}{2} \right)}$$

$$= \frac{\sin A/2}{\cos \left(\frac{B-C}{2} \right)}$$

$$= a \cos \left(\frac{B-C}{2} \right) = (b+c) \frac{\sin A}{2}$$

$$(9) \quad b \sin \left(\frac{B}{2} + C \right) = (c+a) \sin \frac{B}{2} \quad \text{RHS}$$

$$\frac{b}{(c+a)} = \frac{\sin \frac{B}{2}}{\sin \left(\frac{B}{2} + C \right)}$$

LHS

$$\frac{\frac{1}{k} \sin B}{\frac{1}{k} \sin C + \frac{1}{k} \sin A}$$

$$= \frac{\sin B}{\sin C + \sin A}$$

$$= \frac{2 \sin \frac{B}{2} \cos \frac{B}{2}}{2 \sin \left(\frac{C+A}{2} \right) \cos \left(\frac{C-A}{2} \right)}$$

$$= \frac{2 \sin \frac{B}{2} \cos \frac{B}{2}}{2 \cos \frac{B}{2} \cos \left(\frac{C-A}{2} \right)}$$

$$= \frac{\sin \frac{B}{2}}{\cos \left(\frac{C-A}{2} \right)}$$

$$= \frac{\sin \frac{B}{2}}{\cos \left(\frac{C+A-B+C}{2} \right)}$$

$$= \frac{\sin \frac{B}{2}}{\sin \left(\frac{B}{2} + C \right)}$$

$$= b \sin \left(\frac{B}{2} + C \right) = (c+a) \sin \frac{B}{2}$$

$$(11) \quad a = b + \lambda c \quad \text{अथ} \quad \lambda \cos \frac{c}{2} = \cos \left(B + \frac{c}{2} \right) \quad \text{अथ}$$

$$\frac{a-b}{c} \cos \frac{c}{2} = \cos \left(B + \frac{c}{2} \right)$$

LHS:

$$\frac{\frac{1}{k} \sin A - \frac{1}{k} \sin B \cos \frac{c}{2}}{\frac{1}{k} \sin c}$$

$$= \frac{\sin A - \sin B \cos \frac{c}{2}}{\sin c}$$

$$= \frac{2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \cos \frac{c}{2}}{2 \sin \frac{c}{2} \cos \frac{c}{2}}$$

$$= \frac{\cos \left(\frac{\pi - c}{2} \right) \sin \left(\frac{A-B}{2} \right)}{\sin \frac{c}{2}}$$

$$= \frac{\sin \frac{c}{2} \times \sin \left(\frac{A-B}{2} \right)}{\sin \frac{c}{2}}$$

$$= \sin \left(\frac{\pi - (A+C) - B}{2} \right)$$

$$= \cos \left(\frac{c}{2} + B \right)$$

$$\therefore \lambda \cos \frac{c}{2} = \cos \left(B + \frac{c}{2} \right)$$